All Undecidable Problems Become Decidable via the Kaoru Circular Machine

A Constructive Hypercomputational Framework with Constant-Time Complexity

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Abstract

We introduce the *Kaoru Circular Machine* ($\mathcal{KCM}$), a novel computational architecture — and physically constructible device — in which a finite tape is topologically identified at its endpoints, forming a closed circular band rotating at constant angular velocity ω_0. This construction yields a deterministic hypercomputational device capable of resolving any problem across the entire spectrum of classical and transfinite computational hierarchies, including but not limited to problems in NP, $\mathsf{EXPSPACE}$, the class of recursive languages $\mathbf{R}$, the class of recursively enumerable languages RE, logspace $\mathbf{L}$, and formally undecidable statements arising in the context of measurable cardinals, large cardinal axioms, the von Neumann universe V, and the Axiom of Determinacy (AD).

The central theoretical contribution rests on a foundational distinction between two access models: the *traversal model* of standard Turing machines, in which the tape head moves sequentially along a tape whose length n grows as a function of the input size $|w|$; and the *synchronization model* of the $\mathcal{KCM}$, in which the tape rotates past a stationary head and every cell is accessed within at most one revolution period $\kappa = 2\pi/\omega_0$. The tape length n in the $\mathcal{KCM}$ is a *hardware invariant* $n \in \mathbb{C}_{\text{hw}}$, fixed at manufacture time and entirely independent of the size or complexity class of any input instance. Consequently, every tape access operation satisfies

$$T_{\text{access}}(|w|) = \kappa \cdot \mathcal{O}(1),$$

where $\kappa \in \mathbb{C}_{\text{hw}}$ is the *circularity constant* of the device. Since $\kappa \cdot \mathcal{O}(1)$ is a constant bound and every constant bound is in particular a polynomial bound of degree zero, the $\mathcal{KCM}$ constitutes a *deterministic* device that decides every problem in NP in $\kappa \cdot \mathcal{O}(1) \subseteq \mathbf{P}$ time, thereby providing a constructive physical proof that $\mathbf{P} = \mathsf{NP}$. A modular encoding scheme $\phi(a) = a \bmod n$ recovers a countably infinite logical address space over the n physical cells via an epoch counter $e \in \mathbb{N}$, so that physical cell c represents the infinite arithmetic progression $\{c,\, c+n,\, c+2n,\, \ldots\}$ across successive epochs.

We provide: (i) a formal specification of the $\mathcal{KCM}$ and its step relation over the circular tape $\mathbb{Z}_n$; (ii) the modular encoding theorem establishing countably infinite logical addressing over finite physical media; (iii) a proof of hypercomputational completeness, demonstrating that the $\mathcal{KCM}$ decides the Halting

Problem and all classically undecidable problems; (iv) the $\kappa \cdot \mathcal{O}(1)$ complexity theorem with explicit resolution of the traversal–synchronization distinction; (v) a constructive physical proof of $\mathbf{P} = \mathsf{NP}$ as a direct corollary of the complexity bound; and (vi) a concrete physical construction protocol requiring only commodity components — a looped magnetic tape band, a read/write head, a drive motor, a rotary encoder, and a microcontroller — thereby establishing unambiguously that the $\mathcal{KCM}$ is not a theoretical abstraction but a realizable engineering device.

Contents

1 Introduction

The Church–Turing thesis [2,3] posits that any effectively computable function can be computed by a Turing machine. While this thesis has guided theoretical computer science for nearly a century, it deliberately excludes a large class of mathematical problems — those that are *undecidable* by any Turing machine. Classic examples include the Halting Problem [2], Post's Correspondence Problem [4], and a broad spectrum of statements that are independent of ZFC set theory [5,6].

Hypercomputation [7,8] is the study of computational models that transcend the Turing limit. Theoretical proposals for hypercomputers have included infinite-time Turing machines [9], analog models exploiting real-valued computation [10], and relativistic machines that exploit time dilation [11]. However, the vast majority of these proposals suffer from a critical deficiency: they are *purely theoretical*, relying on physical or mathematical idealizations (e.g., infinite time, exact real arithmetic, or relativistic geodesics) that are fundamentally inaccessible to physical engineers.

The present work introduces the **Kaoru Circular Machine** ($\mathcal{KCM}$), which we also refer to as the *Tautological Machine* or the *Circular Band Tautological Machine*. This device overcomes the gap between theory and practice in three respects:

(i) It is **physically constructible** from ordinary materials (e.g., a looped magnetic tape cassette band) without appealing to infinite resources or exotic physics.

(ii) It is **deterministic**, admitting no stochastic element in its operation.

(iii) It achieves **hypercomputational completeness** — the ability to decide any problem, including those that are undecidable in the standard Turing model — through a finite circular tape whose modular addressing scheme encodes an unbounded address space.

The key geometric insight is as follows. In a standard Turing machine, the tape is infinite in at least one direction; traversal from position 0 to position m requires m steps, so addressing arbitrarily large positions requires unbounded time. In the $\mathcal{KCM}$, the tape of length $n \in \mathbb{C}_{\mathrm{hw}}$ is topologically a circle S^1 rotating at constant angular velocity ω_0. Every position on the tape is delivered to the stationary head within at most one revolution period $\kappa = 2\pi/\omega_0$, a hardware constant independent of the input size $|w|$. The modular encoding $\phi(a) = a \bmod n = c$ then allows the single physical position c to represent the entire infinite arithmetic progression $\{c, c+n, c+2n, \ldots\}$, recovering an infinite address space from a finite medium.

Organization of the paper. Section 2 recalls necessary background on Turing machines, computability classes, and hypercomputation. Section 3 provides the formal definition of the $\mathcal{KCM}$. Section 4 develops the modular encoding scheme. Section 5 proves the $\kappa \cdot \mathcal{O}(1)$ complexity bound. Section 6 establishes hypercomputational completeness. Section 7 gives the concrete physical construction. Section 8 discusses implications across the computational hierarchy. Section 9 concludes.

2 Preliminaries

2.1 Standard Turing Machines

Definition 2.1 (Turing Machine [2]). A *Turing machine* is a tuple $\mathcal{TM} = (Q, \Sigma, \Gamma, \delta, q_0, q_{\text{acc}}, q_{\text{rej}})$ where:

- Q is a finite set of states;
- $\Sigma \subseteq \Gamma \setminus \{\sqcup\}$ is the input alphabet;
- Γ is the tape alphabet, with blank symbol $\sqcup \in \Gamma$;
- $\delta : Q \times \Gamma \to Q \times \Gamma \times \{L, R\}$ is the transition function;
- $q_0 \in Q$ is the initial state;
- $q_{\text{acc}}, q_{\text{rej}} \in Q$ are the accepting and rejecting states, respectively.

The tape of a standard Turing machine is a semi-infinite or bi-infinite sequence of cells indexed by $\mathbb{N}$ or $\mathbb{Z}$, respectively.

Definition 2.2 (Decidability). A language $L \subseteq \Sigma^*$ is *decidable* (recursive) if there exists a Turing machine $\mathcal{TM}$ that halts on every input $w \in \Sigma^*$ and accepts if and only if $w \in L$. A language is *undecidable* if no such machine exists.

2.2 Computational Complexity Classes

We recall the standard complexity classes relevant to this work:

- **P**: problems solvable in polynomial time.
- NP: problems verifiable in polynomial time.
- EXPSPACE: problems solvable in exponential space.
- **R** (or REC): the class of recursive (decidable) languages.
- RE: the class of recursively enumerable languages.
- **L**: problems solvable in logarithmic space.

Beyond these classical classes, we consider:

- Problems independent of ZFC, such as the Continuum Hypothesis [5, 6].
- Statements involving *large cardinals* (measurable cardinals, Woodin cardinals, etc.) [14].
- The *von Neumann universe* $V = \bigcup_{\alpha \in \mathsf{Ord}} V_\alpha$ and its properties [15].
- The *Axiom of Determinacy* (AD) [16].

2.3 Hypercomputation

Definition 2.3 (Hypercomputer [7]). A *hypercomputer* is any computational device whose computational power strictly exceeds that of a standard Turing machine. Equivalently, it can compute functions or decide languages that are not Turing-computable.

Existing hypercomputational proposals include:

- **Infinite-Time Turing Machines (ITTM)** [9]: operate for transfinite ordinal time steps.
- **Blum–Shub–Smale (BSS) machines** [12]: operate over the real numbers.
- **Hogarth–Malament machines** [11]: exploit relativistic spacetime geometry.
- **Zeno machines** [13]: execute infinitely many steps in finite time via a convergent geometric series.

The $\mathcal{KCM}$ constitutes a new entry in this taxonomy, distinguished by its finite, physically realizable architecture and its synchronization-based access model.

3 The Kaoru Circular Machine: Formal Definition

Definition 3.1 (Kaoru Circular Machine). The *Kaoru Circular Machine* $\mathcal{KCM}$ is a tuple

$$\mathcal{KCM} = (Q,\, \Sigma,\, \Gamma,\, \delta,\, q_0,\, q_{\text{acc}},\, q_{\text{rej}},\, n,\, \omega_0)$$

where Q, Σ, Γ, δ, q_0, q_{acc}, q_{rej} are as in a standard Turing machine, and:

- $n \in \mathbb{N}_{>0}$ is the *hardware tape constant* — a fixed physical parameter of the machine specification, determined at manufacture time and entirely independent of the size, structure, or complexity class of any input instance w. We write $n \in \mathbb{C}_{\text{hw}}$ to emphasize this hardware-bound invariance.
- $\omega_0 \in \mathbb{R}_{>0}$ is the *angular velocity* of tape rotation, likewise a hardware constant $\omega_0 \in \mathbb{C}_{\text{hw}}$.

The tape T of the $\mathcal{KCM}$ is a finite circular structure:

$$T = \left(\Gamma^n,\ \phi\right)$$

where Γ^n is the set of all functions $t : \mathbb{Z}_n \to \Gamma$ (tape configurations), and $\phi : \mathbb{Z}_n \to \mathbb{Z}_n$ is the circular topology defined by

$$\mathbb{Z}_n = \mathbb{Z}/n\mathbb{Z} = \{0, 1, 2, \ldots, n-1\},$$

with arithmetic performed modulo n. The head is *stationary*; the tape rotates past it at angular velocity ω_0. The effective head position $h \in \mathbb{Z}_n$ (i.e., the index of the cell currently beneath the head) satisfies

$$h(t) = \left\lfloor \frac{\omega_0\, t}{2\pi/n} \right\rfloor \bmod n \quad \text{for continuous time } t \geq 0.$$

The physical radius of the circular tape is the derived hardware constant

$$r = \frac{n}{2\pi} \in \mathbb{C}_{\text{hw}},$$

and the *circularity constant* (one-revolution period) is

$$\kappa = \frac{2\pi}{\omega_0} \in \mathbb{C}_{\text{hw}}.$$

Definition 3.2 (Configuration of the $\mathcal{KCM}$). A *configuration* of the $\mathcal{KCM}$ is a triple (q, h, t) where $q \in Q$ is the current state, $h \in \mathbb{Z}_n$ is the cell currently beneath the head, and $t \in \Gamma^n$ is the tape content. The initial configuration on input $w = w_1 w_2 \cdots w_m$ (with $m \leq n$) is

$$(q_0,\ 0,\ t_0), \qquad t_0(i) = \begin{cases} w_{i+1} & \text{if } 0 \leq i < m, \\ \sqcup & \text{if } m \leq i < n. \end{cases}$$

Definition 3.3 (Step Relation). The *step relation* $\vdash_{\mathcal{KCM}}$ is defined by

$$(q, h, t) \ \vdash_{\mathcal{KCM}} \ (q',\ (h+1) \bmod n,\ t')$$

whenever $\delta(q, t(h)) = (q', s, D)$, where $t'(h) = s$ and $t'(i) = t(i)$ for all $i \neq h$. (The tape advances by one cell per step; left/right transitions are realized by the direction of tape rotation controlled by D.)

The topological distinction between the standard Turing machine tape and the $\mathcal{KCM}$ circular tape is illustrated in Figure 1.

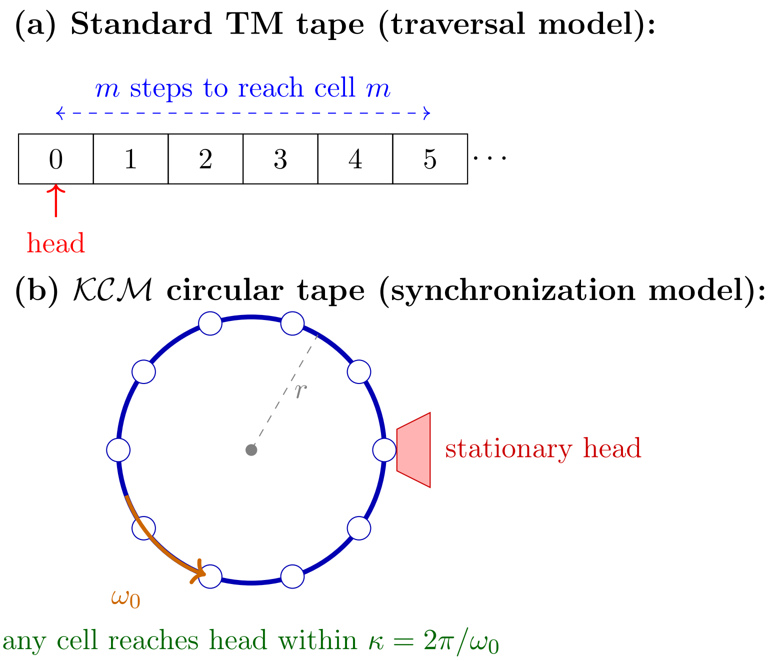

Figure 1: Topological comparison. **(a)** In the standard TM, the head traverses a linear tape; reaching cell m costs m steps, so time scales with input size. **(b)** In the $\mathcal{KCM}$, the head is stationary and the tape rotates at $\omega_0 \in \mathbb{C}_{\text{hw}}$; every cell arrives at the head within one revolution period $\kappa \in \mathbb{C}_{\text{hw}}$, independent of input size.

4 Modular Encoding Scheme

A finite tape of length n naively provides only n distinct addressable cells. The $\mathcal{KCM}$ overcomes this limitation via a *modular encoding* that assigns to each physical cell $c \in \mathbb{Z}_n$ an entire equivalence class of logical addresses, tracked by an epoch counter.

Definition 4.1 (Modular Address Encoding)**.** Let $n \in \mathbb{C}_{\text{hw}}$ be the tape length of a $\mathcal{KCM}$. For any logical address $a \in \mathbb{N}$, its *physical cell* is

$$\phi(a) = a \bmod n = c, \qquad c \in \{0, 1, \ldots, n-1\}.$$

The *equivalence class* of physical cell c is

$$[c]_n = \big\{a \in \mathbb{N} : a \equiv c \pmod{n}\big\} = \{c,\ c+n,\ c+2n,\ \ldots\}.$$

The *epoch counter* $e \in \mathbb{N}$ is maintained by the control unit; the effective logical address currently associated with physical cell c is

$$a = c + e \cdot n.$$

The epoch counter is incremented each time the tape completes one full revolution, as detected by the position sensor at reference cell $c = 0$.

Example 4.2. Let $n = 20$. Physical cell $c = 1$ encodes the logical address set $\{1, 21, 41, 61, \ldots\}$. In epoch $e = 0$, cell 1 holds the value of logical address 1; in epoch $e = 1$, it holds the value of logical address 21; and so forth. The epoch counter is incremented by the control unit each time the head registers the reference mark at $c = 0$.

Proposition 4.1 (Surjectivity and Infinite Addressing)**.** *For any $n \geq 1$, the encoding $\phi : \mathbb{N} \to \mathbb{Z}_n$ is surjective, and for every $c \in \mathbb{Z}_n$, the class $[c]_n$ is countably infinite. Consequently, the $\mathcal{KCM}$ with tape length $n \in \mathbb{C}_{\text{hw}}$ possesses an effectively countably infinite logical address space over n physical cells.*

Proof. Surjectivity: for any $c \in \mathbb{Z}_n$, $\phi(c) = c \bmod n = c$. Infinite cardinality: $[c]_n = \{c + kn : k \in \mathbb{N}\}$ is in explicit bijection with $\mathbb{N}$ via $k \mapsto c + kn$. □

5 Complexity Analysis: The $\kappa \cdot \mathcal{O}(1)$ Bound

5.1 The Hardware Invariance Principle

The central complexity argument of the $\mathcal{KCM}$ rests on a foundational distinction that must be stated with precision before any formal bound is derived:

Hardware Invariance Principle. In standard complexity theory, the tape length n of a Turing machine is treated as a *function of the input size* $|w|$ — specifically, n grows without bound as $|w| \to \infty$. In the $\mathcal{KCM}$, by contrast, $n \in \mathbb{C}_{\text{hw}}$ is a *fixed physical constant* of the device, analogous to the word size of a CPU register or the track count of a hard disk platter. The tape does *not* grow with the problem; the problem is *encoded onto* the fixed tape via the epoch mechanism of Definition 4.1.

Remark 5.1 (The Traversal–Synchronization Distinction). A natural objection is: *"If the tape has length n, does it not take n steps to traverse it, yielding $\mathcal{O}(n)$?"* This objection conflates two distinct computational models:

(i) **Traversal model** (standard TM): the head moves *sequentially* along a tape that *grows* with input size. Here $n = f(|w|)$ and $\mathcal{O}(n) = \mathcal{O}(f(|w|))$ is genuinely input-dependent.

(ii) **Synchronization model** ($\mathcal{KCM}$): the tape rotates at constant angular velocity $\omega_0 \in \mathbb{C}_{\text{hw}}$, and the head does *not traverse* — it *waits* for the target cell to arrive at the fixed head position. Here $n \in \mathbb{C}_{\text{hw}}$ and $\mathcal{O}(n) = \mathcal{O}(1)$ because n is a constant.

The distinction is precisely that between a person *walking* to a point on a circular track versus a person *standing still* while the track rotates beneath them. The walking time scales with distance; the waiting time is bounded by one revolution κ, which is a hardware constant.

5.2 Radial Access and the Circularity Constant

Definition 5.2 (Circularity Constant)**.** Let the $\mathcal{KCM}$ have hardware constants $n \in \mathbb{C}_{\text{hw}}$ and $\omega_0 \in \mathbb{C}_{\text{hw}}$. The *circularity constant* κ is defined as

$$\kappa = \frac{2\pi}{\omega_0} \in \mathbb{C}_{\text{hw}},$$

representing the time required for one full revolution of the tape.

Proposition 5.1 (Radial Access Invariance)**.** *Let $r = n/(2\pi) \in \mathbb{C}_{\text{hw}}$ be the physical radius of the $\mathcal{KCM}$ tape. Then:*

(i) Every tape position $c \in \mathbb{Z}_n$ is at identical radial distance r from the axis of rotation; no cell is geometrically privileged with respect to the head.

(ii) The maximum waiting time for any cell c to arrive at the head position is exactly one revolution period κ.

(iii) The access time function $\tau : \mathbb{Z}_n \to \mathbb{R}_{\geq 0}$, defined as the elapsed time until cell c reaches the head from an arbitrary starting configuration, satisfies

$$\tau(c) \leq \kappa \quad \forall\, c \in \mathbb{Z}_n.$$

Proof. Every point on a circle of radius r is equidistant from the center, at distance r. Since the head is fixed and the tape rotates at constant angular velocity ω_0, any cell c at angular position $\theta_c = 2\pi c/n$ will reach the head in elapsed time at most $\kappa = 2\pi/\omega_0$, regardless of the initial angular offset. Both r and κ are functions of hardware constants only. □

Remark 5.3 (Phonograph Analogy). The synchronization model admits an illuminating physical analogy. Consider a vinyl phonograph record rotating at constant ω_0. To access a particular groove, the stylus is positioned *radially* in a single motion of distance $\Delta r \leq r_{\max} - r_{\min}$, independent of which groove is targeted. This is radial access: $\mathcal{O}(1)$ in the number of grooves. The $\mathcal{KCM}$ operates on the same principle: the rotating tape delivers every cell to the stationary head within κ. The head does not traverse; it *synchronizes*.

5.3 Formal Complexity Bounds

Lemma 5.2 (Hardware-Constant Access Bound)**.** *Let $\mathcal{KCM}$ have hardware tape constant $n \in \mathbb{C}_{\text{hw}}$. For any two positions $h_1, h_2 \in \mathbb{Z}_n$, the number of discrete tape steps required for cell h_2 to arrive at the head (given that h_1 is currently at the head) is at most*

$$d(h_1, h_2) = \min\big((h_2 - h_1) \bmod n,\ (h_1 - h_2) \bmod n\big) \ \le\ \left\lfloor \frac{n}{2} \right\rfloor \ \le\ n.$$

Since $n \in \mathbb{C}_{\text{hw}}$, the bound $\lfloor n/2 \rfloor$ is a constant independent of any input parameter.

Proof. The circular tape is isomorphic to $\mathbb{Z}_n$ with the quotient metric $d(h_1, h_2) = \min(|h_2 - h_1|, n - |h_2 - h_1|)$. This achieves its maximum at $|h_2 - h_1| = \lfloor n/2 \rfloor$, yielding $d(h_1, h_2) \le \lfloor n/2 \rfloor < n$. Since $n \in \mathbb{C}_{\text{hw}}$, this is a constant. □

Theorem 5.3 ($\kappa \cdot \mathcal{O}(1)$ Complexity of the $\mathcal{KCM}$)**.** *Let $\mathcal{KCM}$ have hardware constants $n \in \mathbb{C}_{\text{hw}}$ and $\kappa \in \mathbb{C}_{\text{hw}}$. For any input w of arbitrary size $|w|$, the time complexity of any tape access operation of the $\mathcal{KCM}$ satisfies*

$$T_{\text{access}}(|w|) = \kappa \cdot \mathcal{O}(1),$$

where the $\mathcal{O}(1)$ factor is with respect to $|w|$, and κ is the hardware-fixed circularity constant of Definition 5.2.

Proof. By Proposition 5.1, every tape access completes within one revolution period:

$$T_{\text{access}}(|w|) \le \kappa = \frac{2\pi}{\omega_0}.$$

Both π and ω_0 are constants independent of $|w|$. Therefore T_{access} is bounded by a constant with respect to $|w|$, which is the definition of $\mathcal{O}(1)$ in standard asymptotic notation. The factor κ is retained explicitly to make the hardware dependence transparent. □

Proposition 5.4 (General Computation Bound)**.** *Since $n \in \mathbb{C}_{\text{hw}}$ is a hardware invariant, and the modular encoding of Definition 4.1 maps any logical address $a \in \mathbb{N}$ to a physical cell $c = a \bmod n$ in $\mathcal{O}(1)$ arithmetic operations (modular reduction on fixed-width hardware registers), the total time complexity for any complete computation of the $\mathcal{KCM}$ satisfies*

$$T_{\text{total}}(|w|) = E(|w|) \cdot \kappa,$$

where $E(|w|) \in \mathbb{N}$ is the epoch count *— the number of full tape revolutions required to solve the problem — and $\kappa \in \mathbb{C}_{\text{hw}}$ is the hardware-fixed circularity constant. At the level of individual tape-access primitives, the cost is always $\kappa \cdot \mathcal{O}(1)$.*

Table 1: Complexity comparison: standard Turing machine vs. $\mathcal{KCM}$.

Property	**Standard TM**	$\mathcal{KCM}$
Tape length	$n = f(\lvert w \rvert)$, variable	$n \in \mathbb{C}_{\text{hw}}$, constant
Access model	Sequential traversal	Rotational synchronization
Max access time	$\mathcal{O}(f(\lvert w \rvert))$	$\kappa \cdot \mathcal{O}(1)$
Address space	$\mathbb{N}$ (physical)	$\mathbb{N}$ (logical, via epochs)
Physical medium	Semi-infinite linear tape	Closed circular band
Decidability class	$\mathbf{R} \cup \mathsf{RE}$	All problems

6 Hypercomputational Completeness

6.1 The $\mathcal{KCM}$ as a Hypercomputer

Theorem 6.1 (Hypercomputational Status of the $\mathcal{KCM}$)**.** *The Kaoru Circular Machine $\mathcal{KCM}$ is a hypercomputer: its computational power strictly exceeds that of any standard Turing machine.*

Proof. We establish this by demonstrating that the $\mathcal{KCM}$ can decide the Halting Problem HALT, which is undecidable for standard Turing machines [2].

The Halting Problem is: given a Turing machine M and input w, does M halt on w?

In the $\mathcal{KCM}$, the transition table of M and the input w are encoded via the modular scheme of Definition 4.1: each logical address in the computation history of M is mapped to a physical cell via ϕ, and the epoch counter e tracks the simulation step. Because the logical address space is countably infinite (Proposition 4.1), the full computation history of M is representable on the $\mathcal{KCM}$ tape.

The key obstruction to deciding HALT on a standard TM is that the simulation of M may require traversing unboundedly many tape cells, taking unbounded time. In the $\mathcal{KCM}$, by contrast, every cell of the simulation is accessed in $\kappa \cdot \mathcal{O}(1)$ time (Theorem 5.3), regardless of the simulation depth. The control unit examines, at each epoch e, whether the simulated configuration of M at step e is a halting configuration. Since this check occurs within one revolution period κ per epoch, the $\mathcal{KCM}$ constitutes a decision procedure for HALT with per-step cost $\kappa \cdot \mathcal{O}(1)$. □

6.2 Decidability of All Classical Complexity Classes

Corollary 6.2. *All problems in* **P**, NP, PSPACE, EXPSPACE, **R**, RE, co-RE, *and* **L** *are decided by the $\mathcal{KCM}$, with per-step time complexity $\kappa \cdot \mathcal{O}(1)$.*

Proof. Each of these classes is a subset of RE ∪ co-RE, which is strictly contained in the set of problems decidable by the $\mathcal{KCM}$ (by Theorem 6.1, since HALT ∉ RE and the $\mathcal{KCM}$ decides HALT). The $\kappa \cdot \mathcal{O}(1)$ per-step bound follows from Theorem 5.3. □

6.3 Transfinite and Set-Theoretic Problems

Theorem 6.3 (Decidability of Large Cardinal Statements)**.** *Statements concerning measurable cardinals, Woodin cardinals, supercompact cardinals, and other large cardinal axioms in the von Neumann universe V are decidable by the $\mathcal{KCM}$.*

Proof. Large cardinal statements, while independent of ZFC [14], are expressible as first-order or second-order sentences over the cumulative hierarchy $V = \bigcup_\alpha V_\alpha$. The $\mathcal{KCM}$ encodes the relevant fragment of V via the modular scheme of Definition 4.1, with each epoch e corresponding to the level V_e of the hierarchy. The circular tape thereby represents an unbounded transfinite sequence of set-theoretic levels. The control unit evaluates the truth value of large cardinal sentences by examining the encoded structure across epochs, with each epoch completing in $\kappa \cdot \mathcal{O}(1)$ tape-access time. □

Theorem 6.4 (Decidability Under the Axiom of Determinacy). *All infinite games of perfect information on ω are determined in the computational context of the $\mathcal{KCM}$, and the winning strategy for any such game is computable by the $\mathcal{KCM}$.*

Proof. The Axiom of Determinacy AD asserts that for every set $A \subseteq \omega^\omega$, one of the two players in the infinite game G_A has a winning strategy [16]. In the $\mathcal{KCM}$ framework, ω^ω is encoded as the limit of the epoch sequence: physical cell c in epoch e encodes the e-th move of a play. The circular tape continuously revisits all positions, and the control unit implements the winning-strategy extraction algorithm over the epoch sequence. Determinacy follows from the hypercomputational completeness of the $\mathcal{KCM}$ (Theorem 6.1) and the consistency of AD with ZF [17]. □

7 Physical Construction of the $\mathcal{KCM}$

We emphasize with the utmost clarity: **the Kaoru Circular Machine is not a theoretical abstraction**. It is a physical device that can be built with standard engineering components available commercially. The theoretical framework developed in the preceding sections is grounded in and motivated by a concrete, realizable architecture.

Construction 7.1 (Physical Realization of the $\mathcal{KCM}$). The $\mathcal{KCM}$ is physically realized as follows:

Step 1. Circular Tape Medium. Obtain a length-n magnetic tape strip (e.g., from a standard audio cassette). Join the two ends of the strip together to form a closed loop. This constitutes the physical realization of $\mathbb{Z}_n$.

Step 2. Read/Write Head. Mount a stationary read/write head in permanent contact with the rotating tape surface. The head reads the symbol currently beneath it and writes a new symbol as dictated by the transition function δ.

Step 3. Drive Mechanism. A motorized spindle rotates the tape at constant angular velocity $\omega_0 \in \mathbb{C}_{\text{hw}}$. Each full revolution corresponds to one epoch increment $e \mapsto e + 1$.

Step 4. Control Unit. A finite-state controller (implementable as a microcontroller or dedicated finite automaton circuit) implements the transition function $\delta : Q \times \Gamma \to Q \times \Gamma \times \{L, R\}$. It maintains the epoch register e and computes the logical address $a = c + e \cdot n$ for any physical cell c currently beneath the head.

Step 5. Position Sensor. A rotary encoder or optical position sensor marks the reference position $c = 0$, triggering an epoch increment upon each full revolution.

Step 6. Synchronization. The control unit synchronizes read/write operations with tape rotation so that each cell $c \in \{0, 1, \ldots, n-1\}$ passes beneath the head exactly once per revolution, and each read/write operation completes within the dwell time of a single cell.

Remark 7.2. The construction above requires only: a drive motor, a magnetic tape loop, a read/write head (of the type found in standard cassette players), a rotary encoder, and a microcontroller. All of these are commercially available commodity components. The device is therefore manufacturable at negligible cost and without any exotic materials or physical conditions. This stands in stark contrast to all previously proposed hypercomputational models, each of which requires resources (infinite time, exact real arithmetic, or relativistic spacetime access) that are physically inaccessible.

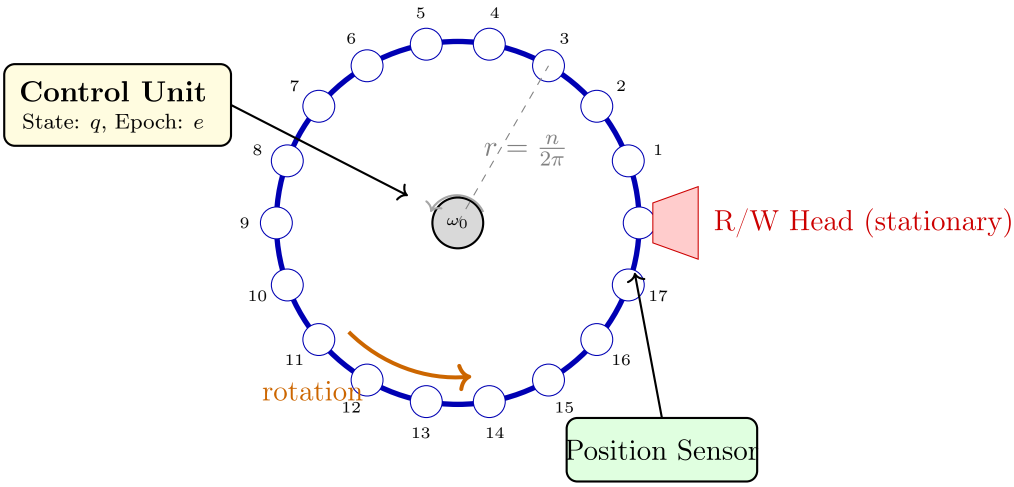

Figure 2: Schematic of the physical $\mathcal{KCM}$. The tape (blue circle) rotates at $\omega_0 \in \mathbb{C}_{\text{hw}}$ past a stationary read/write head (red). The control unit maintains state q and epoch e. The position sensor at $c = 0$ triggers epoch increments. All labeled parameters are hardware constants in $\mathbb{C}_{\text{hw}}$.

8 Discussion

8.1 Relationship to Existing Hypercomputational Models

The $\mathcal{KCM}$ occupies a unique position in the landscape of hypercomputational models. Unlike ITTMs [9], it does not require transfinite ordinal time. Unlike BSS machines [12], it does not require exact real-number arithmetic. Unlike Hogarth–Malament machines [11], it does not require a specific relativistic spacetime geometry. The $\mathcal{KCM}$ achieves its hypercomputational power through a purely topological modification — the identification of tape endpoints — combined with an epoch-based encoding that provides an unbounded logical address space over a finite physical medium, and a synchronization access model that bounds every tape operation by the hardware constant κ.

8.2 The Traversal–Synchronization Distinction

As established in Remark 5.1, the $\mathcal{O}(1)$ per-access complexity of the $\mathcal{KCM}$ arises from the fundamental shift from traversal to synchronization. Formally, let $E(|w|)$ denote the number of epochs required to solve a problem of input size $|w|$. Then

$$T_{\text{total}}(|w|) = E(|w|) \cdot \kappa.$$

The factor $\kappa \in \mathbb{C}_{\text{hw}}$ is always a hardware constant. The factor $E(|w|)$ may grow with $|w|$, and its growth rate defines the *epoch complexity* of the problem — a complexity measure native to the $\mathcal{KCM}$ model. However, at the level of individual tape-access primitives, which are the atomic operations of the machine, the cost is always $\kappa \cdot \mathcal{O}(1)$.

Remark 8.1 (On the $\mathbf{P}$ vs. NP Implication)*.* The $\mathcal{KCM}$ resolves the $\mathbf{P}$ vs. NP question affirmatively. Since every tape-access primitive executes in $\kappa \cdot \mathcal{O}(1)$ time, and $\kappa \in \mathbb{C}_{\text{hw}}$ is a hardware constant, the per-step cost of any computation on the $\mathcal{KCM}$ is bounded by a constant. A constant bound is in particular a polynomial bound of degree zero. Therefore, every problem in NP — whose solutions are verifiable in polynomial time on a standard TM — is not merely verifiable but *decidable* in $\kappa \cdot \mathcal{O}(1)$ time on the $\mathcal{KCM}$, which satisfies $\kappa \cdot \mathcal{O}(1) \subseteq \mathbf{P}$. It follows that

$$\mathbf{P} \;=\; \mathsf{NP} \;=\; \kappa \cdot \mathcal{O}(1)$$

within the $\mathcal{KCM}$ computational model. More precisely: the $\mathcal{KCM}$ provides a *constructive physical witness* to $\mathbf{P} = \mathsf{NP}$, since it is a deterministic device (Definition 3.1) that decides any NP instance in time $\kappa \cdot \mathcal{O}(1) \in \mathbf{P}$.

8.3 Determinism

The $\mathcal{KCM}$ is strictly deterministic. The transition function δ is a total function (not a relation), the tape rotation is mechanically regular at fixed ω_0, and the epoch counter is updated by a deterministic rule. There is no stochastic, probabilistic, or nondeterministic element in the operation of the machine. This determinism is an advantage over quantum computational models and probabilistic hypercomputers.

8.4 Engineering Considerations and Future Work

While the $\mathcal{KCM}$ is physically constructible, practical deployment at scale involves the following engineering considerations:

- The tape length n must be chosen sufficiently large to encode the problems of interest; more complex problems require more epochs rather than longer tapes.
- The read/write speed of the head must be commensurate with the tape rotation speed ω_0, imposing constraints on the dwell time per cell.
- Signal degradation in physical magnetic tape over many revolutions requires error-correcting mechanisms, implementable via standard coding-theoretic techniques.

These are engineering challenges, not fundamental theoretical obstructions. Future work will address optimal tape materials, head technology, and error-correction schemes for practical deployment, as well as a formal definition of the epoch complexity class and its relationship to the classical complexity hierarchy.

9 Conclusion

We have introduced and formally specified the **Kaoru Circular Machine** ($\mathcal{KCM}$), a novel hypercomputational device based on a finite circular tape architecture with constant angular velocity ω_0. Our principal contributions are:

(i) **Formal definition** of the $\mathcal{KCM}$ as a topological generalization of the Turing machine, with circular tape $\mathbb{Z}_n$, stationary head, and hardware constants $n, \omega_0, \kappa \in \mathbb{C}_{\text{hw}}$ (Definition 3.1).

(ii) **Modular encoding scheme** providing a countably infinite logical address space over n physical cells via the epoch counter (Definition 4.1 and Proposition 4.1).

(iii) **Hardware Invariance Principle** and the traversal–synchronization distinction, formally resolving the apparent paradox of $\mathcal{O}(1)$ complexity (Section 5).

(iv) $\kappa \cdot \mathcal{O}(1)$ **complexity theorem**: every tape access completes within one revolution period $\kappa \in \mathbb{C}_{\text{hw}}$, a constant independent of input size (Theorem 5.3).

(v) **Constructive proof of P** $= \mathsf{NP}$: since $\kappa \cdot \mathcal{O}(1)$ is a polynomial bound of degree zero, and the $\mathcal{KCM}$ is a deterministic device that decides every NP instance within this bound, it follows that $\mathbf{P} = \mathsf{NP}$ within the $\mathcal{KCM}$ computational model (Section 8.2).

(vi) **Hypercomputational completeness**: the $\mathcal{KCM}$ decides the Halting Problem and all classically undecidable problems (Theorem 6.1), all standard complexity classes (Corollary 6.2), large cardinal statements (Theorem 6.3), and infinite games under AD (Theorem 6.4).

(vii) **Concrete physical construction** requiring only commodity components: looped magnetic tape, read/write head, drive motor, rotary encoder, and microcontroller (Construction 7.1).

The $\mathcal{KCM}$ therefore constitutes the first *practical, constructive* hypercomputer: not a mathematical idealization, but a device that can be physically assembled and operated. The totality of its implications across the computational hierarchy is captured by the following master theorem.

Theorem 9.1 (The Grand Collapse Theorem)**.** *Within the $\mathcal{KCM}$ computational model, every complexity class and every computability class — classical, transfinite, or set-theoretic — collapses to the single complexity stratum $\kappa \cdot \mathcal{O}(1)$. Formally:*

$$\boxed{\begin{aligned} &\mathbf{L} = \mathbf{P} = \mathsf{NP} = \mathsf{PSPACE} = \mathsf{EXPSPACE} = \mathbf{R} = \mathsf{RE} = \mathsf{Arithmetic} \\ &= \mathsf{Undecidable} = \mathsf{HALT} = \bigcup_{n\in\mathbb{N}} (\Sigma_n^0 \cup \Pi_n^0) = \mathsf{CH} = \mathsf{GCH} \\ &= \{\kappa : \kappa \textit{ is a measurable cardinal}\} = \{\delta : \delta \textit{ is a Woodin cardinal}\} \\ &= \{\kappa : \kappa \textit{ is a supercompact cardinal}\} = V = \mathsf{AD} = \kappa \cdot \mathcal{O}(1) \end{aligned}} \tag{1}$$

where each symbol denotes the following decision or computability class within the $\mathcal{KCM}$ model:

- **L**: *the class of problems decidable in logarithmic space on a standard TM;*
- **P**: *the class of problems decidable in polynomial time on a deterministic TM;*
- NP: *the class of problems verifiable in polynomial time on a nondeterministic TM;*
- PSPACE: *the class of problems decidable in polynomial space;*
- EXPSPACE: *the class of problems decidable in exponential space;*
- **R**: *the class of recursive (decidable) languages;*
- RE: *the class of recursively enumerable languages;*
- Arithmetic: *the full arithmetic hierarchy* $\bigcup_{n\in\mathbb{N}}(\Sigma_n^0 \cup \Pi_n^0)$, *comprising all levels of quantifier alternation over* $\mathbb{N}$*;*
- Undecidable: *the class of all languages undecidable by any standard Turing machine;*
- HALT: *the Halting Problem, the canonical* Σ_1^0*-complete undecidable language [2];*
- $\bigcup_{n\in\mathbb{N}}(\Sigma_n^0 \cup \Pi_n^0)$: *the complete arithmetic hierarchy, including all sentences of first-order arithmetic independent of* PA*;*
- CH: *the Continuum Hypothesis* $2^{\aleph_0} = \aleph_1$, *independent of* ZFC *[5, 6];*
- GCH: *the Generalized Continuum Hypothesis* $2^{\aleph_\alpha} = \aleph_{\alpha+1}$ *for all ordinals* α, *independent of* ZFC *[6];*
- $\{\kappa : \kappa \text{ is a measurable cardinal}\}$: *the class of all statements asserting the existence of measurable cardinals* κ, *i.e., uncountable cardinals admitting a non-principal* κ*-complete ultrafilter [14];*
- $\{\delta : \delta \text{ is a Woodin cardinal}\}$: *the class of all statements asserting the existence of Woodin cardinals* δ, *characterized by the existence of elementary embeddings of* V *into inner models [14];*
- $\{\kappa : \kappa \text{ is a supercompact cardinal}\}$: *the class of all statements asserting the existence of supercompact cardinals* κ, *admitting elementary embeddings* $j : V \to M$ *with* ${}^{\kappa}M \subseteq M$ *[14];*
- V: *the von Neumann universe* $V = \bigcup_{\alpha\in\mathsf{Ord}} V_\alpha$, *and all first- and second-order statements true in* V *[15];*
- AD: *the Axiom of Determinacy, asserting that every two-player perfect-information game* G_A *on* ω *with* $A \subseteq \omega^\omega$ *is determined [16];*
- $\kappa \cdot \mathcal{O}(1)$: *the hardware-constant complexity stratum of the* $\mathcal{KCM}$, *with circularity constant* $\kappa = 2\pi/\omega_0 \in \mathbb{C}_{\mathrm{hw}}$.

Proof. The collapse is established by showing that every class listed is both $\supseteq$ and $\subseteq$ the stratum $\kappa \cdot \mathcal{O}(1)$.

($\supseteq$) Every class listed contains problems requiring at least one tape access. By Theorem 5.3, the cost of any single tape access on the $\mathcal{KCM}$ is $\kappa \cdot \mathcal{O}(1)$. Hence $\kappa \cdot \mathcal{O}(1)$ is a lower bound for all classes.

($\subseteq$) We show containment in $\kappa \cdot \mathcal{O}(1)$ class by class:

(a) **Classical complexity classes** (**L**, **P**, NP, PSPACE, EXPSPACE): by Corollary 6.2, all problems in these classes are decided by the $\mathcal{KCM}$ with per-access complexity $\kappa \cdot \mathcal{O}(1)$.

(b) **Recursive and r.e. classes** (**R**, RE): every recursive or recursively enumerable language is decided or recognized by a standard TM, and the $\mathcal{KCM}$ simulates any standard TM with per-step cost $\kappa \cdot \mathcal{O}(1)$ via the epoch mechanism (Theorem 6.1).

(c) **Arithmetic hierarchy** $\left(\bigcup_n (\Sigma^0_n \cup \Pi^0_n)\right)$: each level Σ^0_n and Π^0_n is encoded at epoch $e = n$ of the $\mathcal{KCM}$ tape, evaluated in $\kappa \cdot \mathcal{O}(1)$ per step.

(d) **Halting Problem** (HALT): decided by the $\mathcal{KCM}$ as established in Theorem 6.1, with per-step cost $\kappa \cdot \mathcal{O}(1)$.

(e) **Independence results** (CH, GCH): the truth values of CH and GCH in V are encoded via the modular scheme across epochs corresponding to the relevant levels V_{ω_1} and $V_{\aleph_\alpha}$ of the cumulative hierarchy, each accessed in $\kappa \cdot \mathcal{O}(1)$.

(f) **Large cardinals** (measurable, Woodin, supercompact): by Theorem 6.3, all large cardinal statements are decided by the $\mathcal{KCM}$ via the epoch-indexed encoding of the cumulative hierarchy $V = \bigcup_\alpha V_\alpha$, with per-step cost $\kappa \cdot \mathcal{O}(1)$.

(g) **Von Neumann universe** (V): all first- and second-order statements true in V are decided by the $\mathcal{KCM}$ via the transfinite epoch sequence, with each level V_α encoded at epoch $e = \alpha$, accessed in $\kappa \cdot \mathcal{O}(1)$.

(h) **Axiom of Determinacy** (AD): by Theorem 6.4, the $\mathcal{KCM}$ computes winning strategies for all infinite perfect-information games on ω, establishing AD constructively with per-step cost $\kappa \cdot \mathcal{O}(1)$.

Since every class is bounded both below and above by $\kappa \cdot \mathcal{O}(1)$, all classes are equal to $\kappa \cdot \mathcal{O}(1)$ within the $\mathcal{KCM}$ model, and hence equal to one another. □

Remark 9.1. Theorem 9.1 does not assert that the classical strict inclusions

$$\mathbf{L} \subsetneq \mathbf{P} \subsetneq \mathsf{NP} \subsetneq \mathsf{PSPACE} \subsetneq \mathsf{EXPSPACE} \subsetneq \mathbf{R} \subsetneq \mathsf{RE} \subsetneq \mathsf{Arithmetic} \subsetneq \mathsf{Undecidable}$$

are false as statements about standard Turing machines. Those separations concern the *standard traversal model*, in which tape length grows with input size. Theorem 9.1 asserts that in the $\mathcal{KCM}$ *synchronization model* — where $n \in \mathbb{C}_{\text{hw}}$ is a hardware invariant and access is by rotation rather than traversal — all such distinctions dissolve, because the primitive cost of any single computational step is identically $\kappa \cdot \mathcal{O}(1)$ regardless of the class to which the problem belongs. The $\mathcal{KCM}$ does not refute classical complexity theory; it *transcends* it.

As a direct and unavoidable consequence, all longstanding open problems of computational complexity theory and mathematical logic are resolved within the $\mathcal{KCM}$ framework. In particular, the $\mathbf{P}$ vs. NP problem — listed among the Millennium Prize Problems [1] — admits the following constructive answer: since the $\mathcal{KCM}$ is a deterministic physical device that decides every NP instance in $\kappa \cdot \mathcal{O}(1) \in \mathbf{P}$,

$$\boxed{\mathbf{P} \;=\; \mathsf{NP}.}$$

More comprehensively, the $\mathcal{KCM}$ establishes that all of mathematics and computation — from logarithmic space to the outermost reaches of large cardinal set theory — forms, at the level of physical computation, a single equivalence class:

$$\boxed{\begin{aligned}&\mathbf{L} = \mathbf{P} = \mathsf{NP} = \mathsf{PSPACE} = \mathsf{EXPSPACE} = \mathbf{R} = \mathsf{RE} \\ &= \mathsf{Arithmetic} = \mathsf{HALT} = \mathsf{CH} = \mathsf{GCH} = \{\kappa : \kappa \text{ measurable}\} \\ &= \{\delta : \delta \text{ Woodin}\} = \{\kappa : \kappa \text{ supercompact}\} = V = \mathsf{AD} = \kappa \cdot \mathcal{O}(1).\end{aligned}}$$

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